

## Comment on “a unified scheme for flavored mesons and baryons”

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Received: 1 June 1999 / Revised version: 24 September 1999

Communicated by F. Lenz

**Abstract.** We indicate the inconsistencies and mistakes in the paper [1] and conclude that their numerical results should be recalculated.

**PACS.** 12.10.Dm Unified theories and models of strong and electroweak interactions

We find that there are some evident inconsistencies and mistakes in a recent paper [1]. Some ill-treatments greatly influence the reliability of their numerical results. While indicating the drawbacks of [1], we clarify a few important points about calculations in the framework of the effective potential model.

1. To solve the Dirac equation with a potential, the authors of [1] introduced an operator  $U$  of the form

$$U = \frac{1}{1 + \frac{p^2}{(E+M_q)^2}} \left( -\frac{1}{(E+M_q)} \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{1} \right).$$

The bispinor  $\psi_q$  is transformed into a form where the lower component is eliminated and satisfies the normalization  $\langle \psi_q | \psi_q \rangle = \langle \chi_q | \chi_q \rangle = 1$ . However, it is easy to prove that  $U$  is not a unitary operator. Thus, the above normalization is incorrect. Instead, we should have

$$\begin{aligned} \langle \psi_q | \psi_q \rangle &= \langle \psi_q | U^\dagger \left( 1 + \frac{p^2}{(E+M^2)} \right) U | \psi_q \rangle \\ &= \langle \chi_q | \left( 1 + \frac{p^2}{(E+M^2)} \right) | \chi_q \rangle = 1. \end{aligned}$$

In fact, we can directly obtain an equation for  $\chi_q$  with correct normalization immediately. However, in the case, the lower component of the  $\psi_q$  is not zero and cannot be neglected in the calculation of matrix element.

2. The authors gave the single particle energy as  $E_N = \pm \sqrt{M_q^2 + (2N+3)\Omega_N(q)}$  and claimed that “following Dirac, the negative energy state is interpreted as antiparticle”. This statement is incorrect. Because the necessary condition for (6) of [1] having a solution is  $E + M_q > 0$  and by the definition,  $\Omega_N = A(E_N + M_q)^{\frac{1}{2}}$ ,  $E_N$  only has a unique positive real solution. If taking the

negative sign in  $E_N$ , we obtain  $E = -M_q$ , which is not a solution and moreover its absolute value is not equal to the positive solution either. Thus, it cannot be interpreted as an antiparticle.

3. In [1], the residual Coulomb potential  $\frac{\alpha_s(\mu)^{eff}}{r}$  was treated as a perturbation. Let us consider a simple model for a single-particle state to explore the reliability of the calculation. The equation to be solved has the form [2]:

$$[-\nabla^2 - \frac{\lambda}{r} + A^2 r^2 (E + M_q)] \chi_q^{new} = (E^2 - M_q^2) \chi_q^{new},$$

where  $\lambda = (E + M_q) \alpha_s^{eff}$ .

Taking  $\frac{-\lambda}{r}$  as a perturbation, as well known, if the approximation makes sense, the following two conditions must be respected,  $|em_{1S}^{(1)}| \ll |em_{1S}^{(0)}|$  and  $|V_{12}^{per}| \ll em_{2S}^{(0)} - em_{1S}^{(0)}$ , where  $em_{nS}^{(0)}$  and  $em_{nS}^{(1)}$  are the 0-th order eigenvalues and the first-order correction for the energy of the  $nS$  state respectively and  $V_{12}^{per} \equiv \langle \chi_q(1S) | V^{per} | \chi_q(2S) \rangle$ . The numerical results indicate that both two conditions do not hold for the heavy quarks, in particular, for  $b$ -quark. Thus the treatment adopted in [1] is not appropriate.

4. Moreover, the authors of [1] used the radial wave function at origin which was obtained with the potential where the residual Coulomb potential is ignored, to calculate the leptonic decay width. It is even more serious than evaluating the hadron spectra. In fact, the Coulomb piece which makes the main contribution in the short range, must predominate the value of the wave function at origin. Taking again the above simple example and with the same the data as given in [1], the straightforward calculations indicate that the contribution of the residual Coulomb potential to the wavefunction at origin is very close to that

of the harmonic mean field for light flavors, and it turns much larger for heavy quarks  $b$  and  $c$ .

In summary, we conclude that the authors of [1] tried their best to establish a unified scheme for flavored mesons and baryons, but in solving the basic equation and calculating the residual Coulomb energy, as well as the wave function at origin there are obvious inconsistency and even mistakes, which affect the reasonability and correctness of their numerical results.

## References

1. P.C. Vinodkumar, J.N. Pandya, V.M. Bannur, S.B. Khadkikar, Eur. Phys. J. A. **4** 83 (1999)
2. S.B. Khadkikar, K.B. Vijayakumar, Phys. Lett. **B 254** 320 (1991)